Team: Russia 2

Saint - Petersburg

Problem №2
A number of Residues

1st Question

Statement 1. $n = 2^m, \quad k = 2^{m+1} - 1 \quad \text{then } T_k \, mod \, n = 0$

$$T_k = \frac{k(k+1)}{2} = \frac{2^{m+1}(2^{m+1}-1)}{2} = 2^m(2^{m+1} - 1)$$

Then $T_k \vdots 2^m$.

Definition. $V_k = T_k \ mod \ n$

Statement 2. For any positive integers l and k in $[1, 2^m - 1]$ $V_l \neq V_k$

Let l and k - some numbers from $[1, 2^m - 1]$ and $l < k$.
Then $l \in [1, 2^m - 2]$ and $k \in [1, 2^m - 1]$.
Consider T_l and T_k:

$$1 \leq T_k \leq 2^{m-1}(2^m - 1)$$

$$1 \leq T_l \leq \frac{(2^m-1)(2^m-2)}{2}$$

Then:

$$0 \leq T_k - T_l \leq 2^m - 1$$

Therefore $V_l \neq V_k$.

Conclusion. There exist just 2^m residues (in other words, n).