Problem 2: **A number of residues**

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1. $u_{2^n} = 2^n$

If $T_k = m$ $(mod\ 2^n)$ then $T_k = m$ $(mod\ 2^{n+1})$ or $T_k = m + 2^n$ $(mod\ 2^{n+1})$. If $T_k = m$ $(mod\ 2^{n+1})$ then $T_{k+2^n} = m + 2^n$ $(mod\ 2^{n+1})$. So $u_{2^{n+1}} = 2u_{2^n}$ and $u_2 = 2$. Then $u_{2^n} = 2^n$.

2. Let p be a prime number and $p \neq 2$ then $u_{p^n} = \frac{p-1}{p}p^{n-1} + u_{p^{n-2}}$

3. $u_m = u_{p_1^{\alpha_1}} \cdot u_{p_2^{\alpha_2}} \cdot \ldots \cdot u_{p_k^{\alpha_k}}$ there $m = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot \ldots \cdot p_k^{\alpha_k}$