Problem 6: Min/Max

Team: Russia

Abstract:

We made values for 1.a, 1.b, 2.a, 2.c

1.a) S is a line.

1) $n > 1$
2) It is simple to see that

$$l_{max} = \sum_{i=1}^{n-1} l_i$$

Where l_i is the distance between the neighbor points.

3) For all i it is right that:

$$l_i \geq l_{min}$$

4) So

$$0 < \frac{l_{min}}{l_{max}} = \frac{l_{min}}{\sum_{i=1}^{n-1} l_i} \leq \frac{l_{min}}{(n-1)l_{min}} = \frac{1}{n-1}$$

5) This case is the same for the situation 2 when the ways lie entirely in the set S.

1.b) S is a circumference.

1) If all *n* points are making the right(which has all sides the same length) polygon l_{max} is the longest diagonal path of the polygon and it is equal to:

$$l_{max} = 2Rsin\frac{\alpha}{2}$$

where α is equal to:

$$\alpha = \frac{2\pi}{n}[\frac{n-1}{2}]$$

2) And the l_{min} is the side of the polygon and is equal to:

$$l_{min} = 2Rsin\frac{\alpha}{2}$$

where α is equal to:

$$\alpha = \frac{2\pi}{n}$$

3) So

$$\frac{l_{min}}{l_{max}} = \frac{\sin\frac{\pi}{n}}{\sin\frac{\pi}{n}[\frac{n-1}{2}]}$$

4) If we move even one point , we make l_{min} , smaller, l_{max} is getting bigger, so the ration is getting smaller, that is why

$$\frac{l_{min}}{l_{max}} \leq \frac{\sin\frac{\pi}{n}}{\sin\frac{\pi}{n}[\frac{n-1}{2}]}$$

2.c) S is the boundary of a convex polygon.

1) This case is the same as 1.b because in 1.b the distance is calculated the same way: by the sides of the polygon.