

# Problem 4: A Baby Chess

## Team: Russia

### Abstract:

- 1) For the knight on the board  $2 \times n$  we proved following statements:
  - 1) If the knight is placed so that the distance along the long side of the board from this square to both ends of the board is  $4m + 2$  or  $4m + 3$  (where  $m \in \mathbb{N}$ ), player, who is the first to move the knight, wins wherever he moves the knight.
  - 2) If the knight is placed so that the distance along the long side of the board from this square to both ends of the board is not  $4m + 2$  or  $4m + 3$  (where  $m \in \mathbb{N}$ ), player, who is the first to move the knight, loses wherever he moves the knight.
  - 3) If the knight is placed so that the distance along the long side of the board from this square to one end is  $4m + 2$  or  $4m + 3$  (where  $m \in \mathbb{N}$ ), and to the other end of the board is not  $4m + 2$  or  $4m + 3$  (where  $m \in \mathbb{N}$ ), player, who is the first to move the knight, wins by following the winning strategy, which was introduced by us.
- 2) For the king on the board  $1 \times n$  we proved following statements:
  - 1) If the king is placed so that there are two or one blocks of squares each of odd amount of squares, the player, who is the first to move the king, wins wherever he makes the first move.
  - 2) If the king is placed so that there are two or one blocks of squares each of even amount of squares, the player, who is the first to move the king, loses wherever he makes the first move.
  - 3) If the king is placed so that there are two blocks of squares one of which consists of even amount of squares, and another of odd amount of squares, the player, who is the first to move the king, wins by following the rules of winning strategy, which was introduced by us.
- 3) For the king on the board  $m \times n$ , where  $m \neq 1$  and  $n \neq 1$ , we proved following statements:
  - 1) If  $m \cdot n$  is even, the player, who is the first to move the king, wins, when he follows the winning strategy, which was introduced by us.
  - 2) If  $m \cdot n$  is odd, the player, who is the second to move the king, wins, when he follows the winning strategy, which was introduced by us.
- 4) For the rook on the board  $m \times n$  we proved following statements:
  - 1) If  $m \cdot n$  is even, the player, who is the first to move the rook, wins, when he follows the winning strategy, which was introduced by us.
  - 2) If  $m \cdot n$  is odd, the player, who is the second to move the rook, wins, when he follows the winning strategy, which was introduced by us.
- 5) For the rook with the trace we proved following statements:
  - 1) Player, who is the first to move the rook, wins by following the winning strategy, which was introduced by us.
- 6) For the bishop on the board  $2 \times n$  we proved following statements:
  - 1) If the bishop is placed so that there are two or one blocks of squares each of odd amount of squares, the player, who is the first to move the bishop, wins wherever he makes the first move.
  - 2) If the bishop is placed so that there are two or one blocks of squares each of even amount of squares, the player, who is the first to move the bishop, loses wherever he makes the first move.
  - 3) If the bishop is placed so that there are two blocks of squares one of which consists of even amount of squares, and the other of odd amount of squares, the player, who is the first to move the bishop, wins by following the rules of winning strategy, which was introduced by us.

- 7) For the bishop on the board  $m \times n$ , where  $m > 2$  and  $n > 2$ , we proved the following statements:
- 1) If  $m \cdot n$  can be divided by 4 entirely (meaning that the amount of squares the same color as the bishop is even), the player, who is the first to move the bishop, wins by following the winning strategy.
  - 2) If  $m \cdot n$  can not be divided by 4 entirely (meaning that the amount of squares the same color as the bishop is odd), the player, who is the second to move the bishop, wins by following the winning strategy.
- 8) For the bishop with the trace on the board  $m \times n$ , where  $m > 2$  and  $n > 2$ , we proved the following statements:
- 1) Player, who is the first to move the bishop, wins by following the rules of winning strategy, which was introduced by us.

## Knight

Let's notice that there is no difference between playing white or black knight.

### Knight on the board $2 \times n$

This situation is the same for knight and for knight with trace.

This kind of board has its own interest to be considered, because with different first coordinates different players win. The ending of the game depends on the first move of the player, who is the first to move the knight, because since then the students do not have any opportunities to move the knight and they are forced to go along the long side of the board on the only square they can go to.

For example, the board  $2 \times 9$ :

- 1) When the knight is placed on the square with coordinates (3,1), the player, who is the first to move the knight, wins wherever he first moves the knight.

By red color is marked the square, that the bishop is originally placed. By 1 are marked moves of the player, who is the first to move the bishop. By 2 are marked moves of the player, who is the second to move the bishop.

1				1				1
						2		

- 2) When the knight is placed on the square with coordinates (1,1) or (5,1), the player, who is the first to move the knight, loses wherever he first moves the knight.

		1				1		
				2				2

		1				1		
2								2

- 3) When the bishop is placed on the square with coordinates (4,1), the winner is depended on where the first move is done: if the first move is done to the side, where there are  $4m$  or  $4m - 1$  (where  $m \in \mathbb{N}$ ) amount of squares, the player, who was the first to move the bishop, wins; if

the first move is done to the side, where there is another amount of squares, the player, who was the first to move the bishop, loses.

	1				1			
							2	

Let's find the winning strategy for the player1.

Let's define the 'winning strategy':

The strategy is considered to be the winning, if the player, who is following this strategy, wins wherever the other player moves the knight.

The rules of the winning strategy:

- 1) Player1 has to make his moves so that he would leave an even amount of moves of the knight to the end of the board. This means that to the end of board there should be left after player1's move  $4m$  or  $4m + 1$  (where  $m \in \mathbb{N}$ ) number of squares in projection on the long side of the board.

If the player has no opportunity to make the first move that follows the rule #1, it means that he does not have chances to win at all.

To sum up:

- 1) If the knight is placed so that the distance along the long side of the board from this square to both ends of the board is  $4m + 2$  or  $4m + 3$  (where  $m \in \mathbb{N}$ ), player, who is the first to move the knight, wins wherever he moves the knight.
- 2) If the knight is placed so that the distance along the long side of the board from this square to both ends of the board is not  $4m + 2$  or  $4m + 3$  (where  $m \in \mathbb{N}$ ), player, who is the first to move the knight, loses wherever he moves the knight.
- 3) If the knight is placed so that the distance along the long side of the board from this square to one end is  $4m + 2$  or  $4m + 3$  (where  $m \in \mathbb{N}$ ), and to the other end of the board is not  $4m + 2$  or  $4m + 3$  (where  $m \in \mathbb{N}$ ), player, who is the first to move the knight, wins by following the winning strategy.

# King

Let's notice that there is no difference between playing white or black king.

Let's notice that both conditions of the problem are equal for the king, because he has no trace and he also can not pass through already visited squares at all.

## King on the board 1xn

This kind of board has its own interest to be considered, because with different first coordinates different players win. The ending of the game depends on the first move of the player, who is the first to move the king, because since then the students do not have any opportunities to move the king and they are forced to go along the long side of the board on the only square they can go to.

For example, the board 1x7:

- 1) When the king is placed on the square with coordinates (4,1), the player, who is the first to move the king, wins wherever he first moves the king.

By red color is marked the square, that the king is originally placed. By 1 are marked moves of the player, who is the first to move the king. By 2 are marked moves of the player, who is the second to move the king.



- 2) When the king is placed on the square with coordinates (3,1), the player, who is the first to move the king, loses wherever he first moves the king.

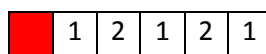


The board 1x6:

- 1) When the king is placed on the square with coordinates (3,1), the winner is depended on where the first move is done: if the first move is done to the side, where there is odd amount of squares, the player, who was the first to move the king, wins; if the first move is done to the side, where there is even amount of squares, the player, who was the first to move the king, loses.



- 2) When the king is placed on the square with coordinates (1,1), the player, who is the first to move the king, wins wherever he first moves the king.



Let's find the winning strategy for the player1.

Let's define the 'winning strategy':

The strategy is considered to be the winning, if the player, who is following this strategy, wins wherever the other player moves the king.

Let's define 'the block':

The set of squares is called 'the block', if it consists only of squares that the player can move the king to only from other squares of 'the block'.

The rules of the winning strategy:

- 1) Player1 has to leave after his first move a block of even amount of squares or to move to the square that has no squares else to go to for his first move. That is why the other player will leave a block of odd amount of squares after him, and player1 will leave a block of even amount of squares, at the end the other player will leave a block of only one square. Player1 will move the king to this square, that has no squares left to go, and will win.

If the player has no opportunity to make the first move that follows the rule #1, it means that he does not have chances to win at all.

To sum up:

- 4) If the king is placed so that there are two or one blocks of squares each of odd amount of squares, the player, who is the first to move the king, wins wherever he makes the first move.
- 5) If the king is placed so that there are two or one blocks of squares each of even amount of squares, the player, who is the first to move the king, loses wherever he makes the first move.
- 6) If the king is placed so that there are two blocks of squares one of which consists of even amount of squares, and another of odd amount of squares, the player, who is the first to move the king, wins by following the rules of winning strategy.

## King on the board mxn

Let's find the winning strategy for the player1.

Let's define the 'winning strategy':

The strategy is considered to be the winning, if the player, who is following this strategy, wins wherever the other player moves the king.

Let's define 'the block':

The set of squares is called 'the block', if it consists only of squares that the player can move the king to only from other squares of 'the block'.

The rules of the winning strategy:

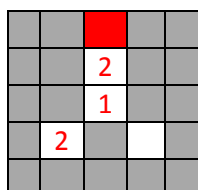
- 1) Player1 has to leave after his moves only blocks of even amount of squares or to move to the square that has no squares else to go to for his first move. That is why the other player will leave blocks of odd amount of squares after him, that at the end will be a block of only one square. Player1 will move the king to this square, that has no squares left to go, and will win.

If the player has no opportunity to make a move that follows the rule #1, it means that the previous turn he also did not follow this rule. If he had no opportunity else to go then it means that he did not follow this rule the previous turn too. That can lead to the fact that he did not follow rule #1 at his first move. This means that he had no ability to follow the rules at the first move, so he did not have any chances to win at all.

- 2) Player1 has to make his moves so that the square he goes to has more visited squares in neighborhood than other squares he can go to (the edge of the board should also be considered as the borders of visited squares). But this rule is under the rule #1, meaning that if the moving to the square with the biggest number of visited neighbors will lead to player1's defeat, he has to move following the rule #1 at first.

That is why there would not be any situations when a block of even amount of squares is divided into two blocks of odd amount of squares. Player1 has left a block of even amount of squares, but he loses.

By the color red is marked the square, that the king stays at. By grey color are marked the squares, that the king has already visited. By 1 is marked moves of the player1. By 2 is marked moves of the player2.



1) Example how to follow the strategy rules:

	2		

2) The right move of player1 following the rules of the winning strategy and his victory (there are alternative ways for the player1 to go, but there is no difference in the end):

(1)		1	
	2		

	2	1	
	2		

1	2	1	
	2		

3) The wrong move of the player2 not following the winning strategy and his defeat:

	1		
	2		

(2)	1	2	
	2		

To sum up:

- 1) If  $m \cdot n$  is even, the player, who is the first to move the king, wins, when he follows the winning strategy.
- 2) If  $m \cdot n$  is odd, the player, who is the second to move the king, wins, when he follows the winning strategy.

# Rook

## Rook on the board $m \times n$

Let's notice that there is no difference between playing white or black rook.

Let's find the winning strategy for the player1.

Let's define the 'winning strategy':

The strategy is considered to be the winning, if the player, who is following this strategy, wins wherever the other player moves the rook.

Let's define 'the block':

The set of squares is called 'the block', if it consists only of squares that the player can move the rook to only from other squares of 'the block'.

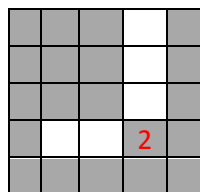
The rules of the winning strategy:

- 1) Player1 has to leave after his move only blocks of even amount of squares or to move to the square that has no squares else to go to. That is why the other player will leave blocks of odd amount of squares after him, that at the end will be a block of only one square. Player1 will move the rook to this square, that has no squares left to go, and will win.

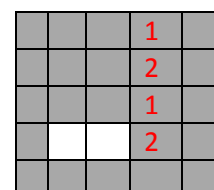
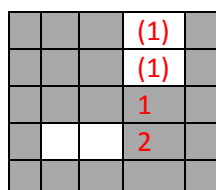
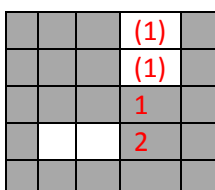
If the player has no opportunity to make a move that follows the rule #1, it means that the previous turn he also did not follow this rule. If he had no opportunity else to go then it means that he did not follow this rule the previous turn too. That can lead to the fact that he did not follow rule #1 at his first move. This means that he had no ability to follow the rules at the first move, so he did not have any chances to win at all.

- 1) Example how to follow the strategy rules:

By grey color are marked squares, that the rook has already visited. By 1 are marked moves of the player1. By 2 are marked moves of the player2.

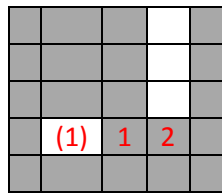


- 2) The right move of player1 following the rules of the winning strategy and his victory (there are alternative ways for the player1 to go, but there is no difference in the end):



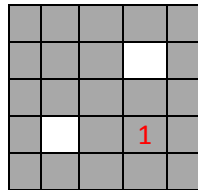


- 3) The wrong move of the player2 not following the winning strategy and his defeat (there are alternative ways for the player1 to go, but there is no difference in the end):

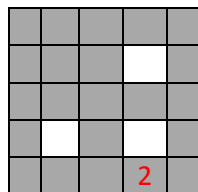


But:

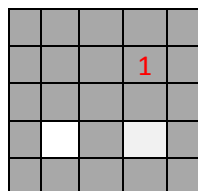
- 1) There are some situations when player1 leaves even amount of squares but he loses. That is when even amount of squares is divided into two blocks of odd amount of squares.



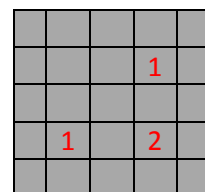
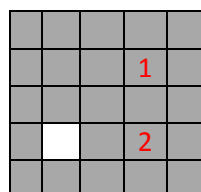
- 2) It is obvious that the turn before player1 was staying on one of the horizontal and vertical lines on the intersection of which he is staying now. Let's think he was staying like it is shown on the picture below (there is no difference of where he was actually staying):



- 3) Then, following the winning strategy player1 would move the rook as it is shown on the picture below:



- 4) And player1 wins:



To sum up:

- 1) If  $m \cdot n$  is even, the player, who is the first to move the rook, wins, when he follows the winning strategy.
- 2) If  $m \cdot n$  is odd, the player, who is the second to move the rook, wins, when he follows the winning strategy.

## Rook with trace on the board $m \times n$

Let's find the winning strategy for the player1. Let's consider that player1 is the first to move the rook.

The main rule:

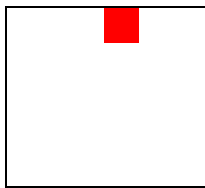
- 1) Player1 moves the rook so that he does not leave after him a straight line, the end of which has no squares to go to from.
- 2) If the square he wants to go to is like in the main rule he has to move the rook to the square that is previous to the square he wants to go to.

**Case #1:** The rook is placed on the square, which is on the edge of the board

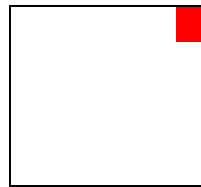
There are two types of such squares:

By red color is marked the square, that the rook is originally placed.

Type #1



Type #2



- 1) Player1 chooses the direction of all his moves: along the long side of the board.
- 2) The first move player1 makes to the nearest end of the line, which follows the chosen direction. When  $m = n$  player1 makes his moves in the direction, which is perpendicular to the side of the board he stays at. Then the board can have these shapes:

#1



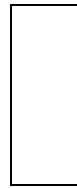
or

#2

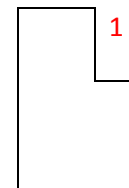


+

#3

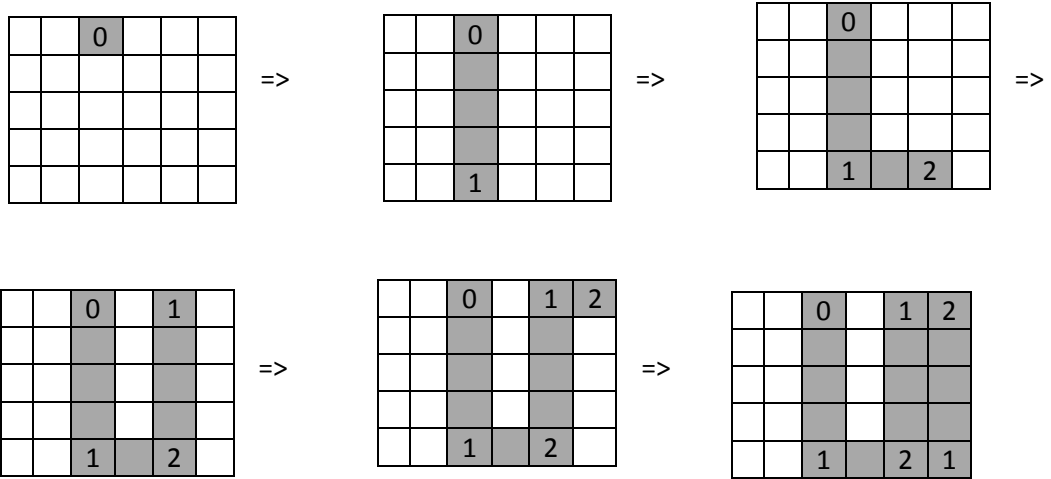


or



- 3) Player2 has to make his move in the perpendicular direction then player1 makes a move to the end of line following the chosen direction. Player1 will win or the board will have the shape #1 or #2 or #3. Player2 has to make a move in the perpendicular direction. And so on and so on.
- 4) In the end player2 will move the rook to a straight line parallel to the direction that player1 had chosen, the end of which has no squares to go from. So player1 will go the end of this line and will win.

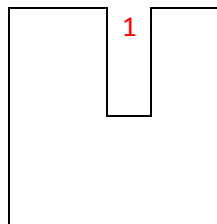
5) Example:



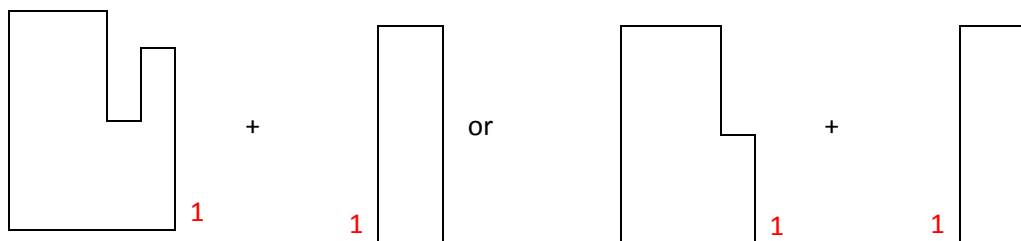
Case #2: The rook is placed on the square, which is not on the edge of the board

The rules of the winning strategy:

- 1) Player1 chooses the direction of his moves: along the long side of the board.
- 2) The first move player1 makes to the nearest end of the line, which follows the chosen direction. Then the board can have these shapes:



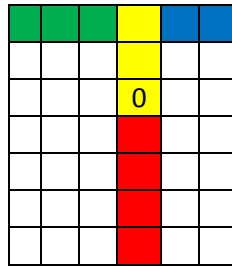
- 3) Player2 has to make his move in the perpendicular direction then player1 makes a move to the end of line following the chosen direction. Then the board can have these shapes:



- 4) If player 1 moves to the quadrangle, the situation becomes similar to the case #1 and player1 wins.
- 5) If player2 moves to the side with the 'ledge' the board shape can be different:

### First board shape

- 1) If player2 moves the rook to the line, which does not include the 'ledge', player1 will pass the whole line and the situation will go like in paragraph, until there would be only one square above from the edge of the board to the 'ledge' in its direction.
- 2) If player2 could move the rook to this square, he would win or player1 would not be able to move in the chosen direction, so he would move in perpendicular direction, he would move the rook to a part of board shaped like quadrangle and he would be moving along the short side, so like in case #1 player2 would win.
- 3) The goal of player1 is to prevent player2 from going there.
- 4) Let's make some conventional signs:



The number of green squares let's mark as 't'. The number of blue squares let's mark as 'k'. The number of yellow squares let's mark as 'x' (the 'ledge'). The number of red squares let's mark as 'y'.  $m \geq n$ .

$$5) \min(t, k) \leq \frac{(n-1)}{2} \leq \frac{(m-1)}{2} \leq y$$

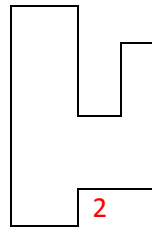
- 6) The number of crossing the red squares by player2 (player1 will not pass these squares because of his strategy) let's mark as 'c'.

$$c \leq \min(t, k)$$

- 7) So there would be only one square on the line that includes the 'ledge' (let's mark it as 'G'), only if player2:
  - 1) Makes his first move to the furthest edge of the board
  - 2) Moves the rook to the end of the line, that is parallel with the short side of the board, when he is on the part of the board which is not divided by the 'ledge'
  - 3) Moves the rook to the neighbor square by the short side of the board when he stays on the part of the board which is divided into two pieces by the 'ledge'
- 8) In other cases he would cross the red squares less than  $(y-1)$  times. Therefore, he would not have any chances to move the rook to the G-square and he will not win. So he has to follow this strategy.
- 9) In this case player1 has to follow the main rule and to move the rook not to the square, from which player2 will go to the G-square, he has to move to the previous square. So player2 will have two options: to move to the square, from which player1 will go to the G-square and player2 will lose; or to move the rook in the direction that is parallel with the short side of the board, so player2 will move the rook to the line that is parallel to the long side of the board, which borders with the 'ledge', so player2 will make a board which is one square wide and the end of which has no squares else to go. Player1 will pass this whole line and win.
- 10) So player1, who is making the first move, wins by following this strategy.

## Second board shape

- 1) If player2 moves the rook to the line which includes the 'ledge', the board would have this shape:



- 2) Player1 would pass this whole line in the chosen direction if the next move of player2 to one of the end of the line (horizontal, in this case) will not split the board into boards which have the long side parallel with the move of player2, which will lead to the player1's defeat if player1 will follow the rules of case #1. Example of this situation:

			2		1
2			1		
			0		
			1		2

- 3) So player1 would make such move if  $\max(k, t) \leq x$ .
- 4) If  $\max(k, t) \geq x$  player1 would move the rook following the chosen direction to the biggest number of squares 's', where 's' follows these conditions:
- 1)  $s \geq \max(k, t)$
  - 2)  $m - s - 2 \geq \max(k, t)$
- 5) If player1 can make a move following paragraph 1 or 2, and if player2 will move from the line that includes the 'ledge', player1 would win, because player2 would move the rook along the short side of new board.
- 6) If player2 makes a move to a square of the line that includes the 'ledge':
- 1) Because 's' is the biggest number of squares that follows first or second condition, moving even to one square would short the number of squares to the square where the rook was originally placed.
  - 2) Player1 would change the direction of his moves and would move the rook to that end of the line, where the length of his move (in squares) would be bigger than each distance from the square he goes to to the short side of the board.
- 7) Following these strategy player1, who was the first to move the rook, wins.

To sum up:

- 1) Player, who is the first to move the rook, wins by following this strategy.

# Bishop

Let's notice that there is no difference between playing white or black bishop.

## Bishop on the board 1xn

Nobody wins on this kind of board, because no moves can be done.

## Bishop on the board 2xn

This situation is the same for bishop and for bishop with trace.

This kind of board has its own interest to be considered, because with different first coordinates different players win. The ending of the game depends on the first move of the player, who is the first to move the bishop, because since then the students do not have any opportunities to move the bishop and they are forced to go along the long side of the board on the only square they can go to.

For example, the board 2x7:

- 4) When the bishop is placed on the square with coordinates (2,2), the player, who is the first to move the bishop, wins wherever he first moves the bishop.

By red color is marked the square, that the bishop is originally placed. By 1 are marked moves of the player, who is the first to move the bishop. By 2 are marked moves of the player, who is the second to move the bishop.

			2		2	
1		1		1		1

- 5) When the bishop is placed on the square with coordinates (3,1), the player, who is the first to move the bishop, loses wherever he first moves the bishop.

	1		1		1	
2				2		2

The board 2x8:

- 1) When the bishop is placed on the square with coordinates (3,2), the winner is depended on where the first move is done: if the first move is done to the side, where there is odd amount of squares, the player, who was the first to move the bishop, wins; if the first move is done to the side, where there is even amount of squares, the player, who was the first to move the bishop, loses.

2				2		2	
	1		1		1		1

- 2) When the bishop is placed on the square with coordinates (1,2), the player, who is the first to move the bishop, wins wherever he first moves the bishop.

		2		2		2	
	1		1		1		1

Let's find the winning strategy for the player1.

Let's define the 'winning strategy':

The strategy is considered to be the winning, if the player, who is following this strategy, wins wherever the other player moves the bishop.

Let's define 'the block':

The set of squares, the same color as bishop is, is called 'the block', if it consists only of squares that the player can move the bishop to only from other squares of 'the block'.

The rules of the winning strategy:

- 1) Player1 has to leave after his first move a block of even amount of squares or to move to the square that has no squares else to go to for his first move. That is why the other player will leave a block of odd amount of squares after him, and player1 will leave a block of even amount of squares, at the end the other player will leave a block of only one square. Player1 will move the bishop to this square, that has no squares left to go, and will win.

If the player has no opportunity to make the first move that follows the rule #1, it means that he does not have chances to win at all.

To sum up:

- 7) If the bishop is placed so that there are two or one blocks of squares each of odd amount of squares, the player, who is the first to move the bishop, wins wherever he makes the first move.
- 8) If the bishop is placed so that there are two or one blocks of squares each of even amount of squares, the player, who is the first to move the bishop, loses wherever he makes the first move.
- 9) If the bishop is placed so that there are two blocks of squares one of which consists of even amount of squares, and another of odd amount of squares, the player, who is the first to move the bishop, wins by following the rules of winning strategy.

## Bishop on the board $m \times n$

Let's find the winning strategy for the player1.

Let's define the 'winning strategy':

The strategy is considered to be the winning, if the player, who is following this strategy, wins wherever the other player moves the bishop.

Let's define 'the block':

The set of squares, the same color as bishop is, is called 'the block', if it consists only of squares that the player can move the bishop to only from other squares of 'the block'.

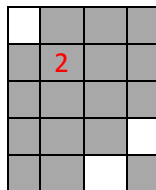
The rules of the winning strategy:

- 2) Player1 has to leave after his moves only blocks of even amount of squares or to move to the square that has no squares else to go. That is why the other player will leave blocks of odd amount of squares after him, that at the end will be a block of only one square. Player1 will move the bishop to this square, that has no squares left to go, and will win.

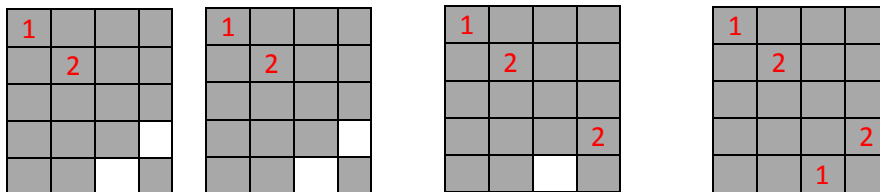
If the player has no opportunity to make a move that follows the rule #1, it means that the previous turn he also did not follow this rule. If he had no opportunity else to go then it means that he did not follow this rule the previous turn too. That can lead to the fact that he did not follow rule #1 at his first move. This means that he had no ability to follow the rules at the first move, so he did not have any chances to win at all.

- 4) Example how to follow the strategy rules:

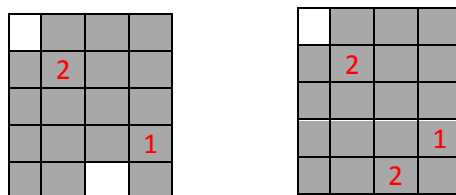
By grey color are marked squares, that the bishop has already visited and that he can not go to. By 1 are marked moves of the player1. By 2 are marked moves of the player2.



- 5) The right move of player1 following the rules of the winning strategy and his victory:



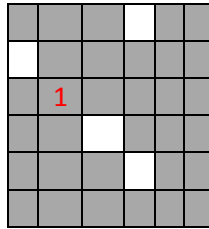
- 6) The wrong move of the player2 not following the winning strategy and his defeat:



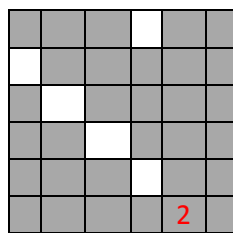


But:

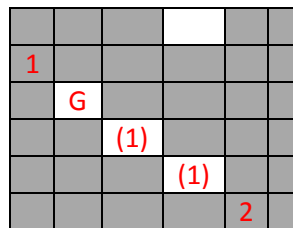
- 1) There are some situations when player1 leaves even amount of squares but he loses. That is when even amount of squares is divided into two blocks of odd amount of squares.



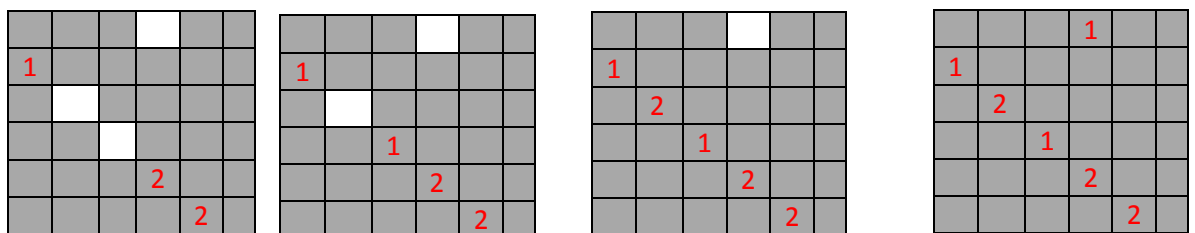
- 2) It is obvious that the turn before player1 was staying on one of the diagonal lines on the intersection of which he is staying now. Let's think he was staying like it is shown on the picture below (there is no difference of where he was actually staying):



- 3) Then, following the winning strategy player1 would move the bishop as it is shown on the picture below (there are some alternative ways he could do it):



- 4) If player2 moves to the G-square, player1 wins in one move. If player2 moves on other free squares, player1 moves to the pre-last square of this diagonal line, player2 has no choices, only to move to the G-square, and then player1 wins.



To sum up:

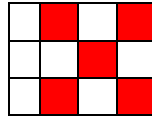
- 1) If  $m \cdot n$  can be divided by 4 entirely (meaning that the amount of squares the same color as the bishop is even), the player, who is the first to move the bishop, wins, when he follows the winning strategy.
- 2) If  $m \cdot n$  can not be divided by 4 entirely (meaning that the amount of squares the same color as the bishop is odd), the player, who is the second to move the bishop, wins, when he follows the winning strategy.

## Bishop with trace on the board mxn

Let's find the winning strategy for the player1. Let's consider that player1 is the first to move the bishop.

Let's define the 'main diagonal':

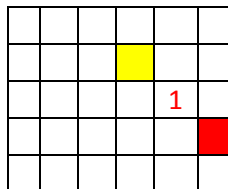
A diagonal line is considered to be the main diagonal if it contains the angle square. Example (By red color are marked the squares of main diagonals):



The rules of the winning strategy:

The main rule:

- 1) Player1 moves so that he does not leave after him a straight diagonal line, the end of which has no squares to go to from.
- 2) If the square he wants to go to (painted in yellow on the picture) is like in the main rule he has to move the bishop to the square that is previous to the square he wants to go to or the next if he can not go to the previous.

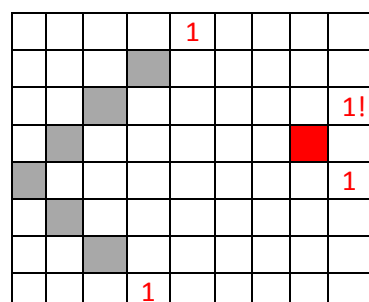
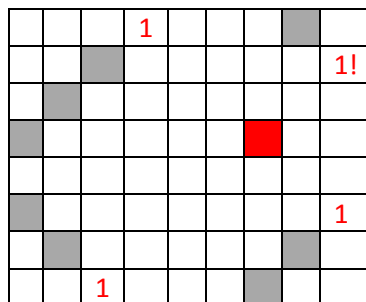


First move rules:

- 1) If the bishop is originally placed on the main diagonal player1 has to make his first move to the angle square and win in one move.
- 2) Player1 has to choose a direction to move: he has to choose an angle of the board so, that when he moves the bishop to the edge of the board in this direction the other player has smaller number of squares to go to than if player1 went to another angles.
- 3) Player1 has to follow this direction and to go to the angle he had chosen. But if he stands on the diagonal line that is along the chosen direction and there is only one square to the edge of the board he has to move the bishop to this square.

Example how to choose the direction and make the first move:

By red color is marked the square, that the bishop is originally placed. By grey color are marked the squares, where player2 can go after player1's first move. By 1 are marked possible moves of the player1. By 1! is marked the right first move of player1.



Player2's first move:

- 1) Player2 can only move to one side from the direction chosen by player1. Player2 moves to another diagonal line that is parallel to the player1's first move's direction.

Player1's second move:

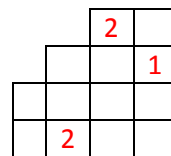
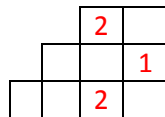
- 3) Player1 goes to the end of this diagonal that in the opposite direction of his first move's direction. Or he follows the rule #3 if the situation is like in rule #3.

Player2's second move:

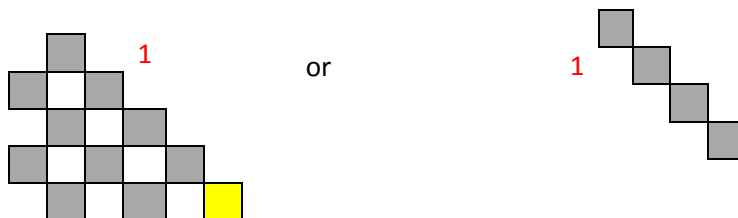
- 1) Then there can be some alternative situations.

Situation 1:

- 1) If player2 in his first move has gone to the neighbor diagonal line to the line that player1 made his first move along, then player1 has to follow rule #3.
- 2) If player2 makes all his next moves to the neighbor diagonal line to the line that player1 made his previous move along, then from some time player2 will move the bishop to the main diagonal and player1 will win by rule #1, or player2 will move to the other side of the board.



- 3) Player1 will go this whole diagonal line to the end of it in the opposite direction to his first move. Player2 will be able only to go to a board having this shape:

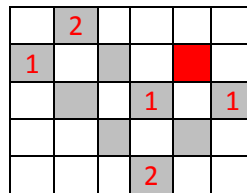


- 4) If he moves to the board with the first shape, wherever he moves the bishop player1 goes along his first move's direction to the square that is on the edge of the board. Player2 will lose or will have to go to another diagonal line that is parallel to the player1's first move's direction. Player1 will make another move along his first move's direction to the square that is on the edge of the board, so he will force player2 to move to the diagonal line which is parallel to the player1's first move's direction and is in neighborhood with diagonal edge of this shape-board. When player2 will move to this diagonal line player1 will move the bishop to the square that is painted in yellow on the picture and win the game.
- 5) Player1 would not go to the square which is marked by '1' on first shape board if this is one of the squares of the main diagonal (see main rule). So he will move the bishop to the previous square. If player2 will go along player1's previous turn, he will move to the main diagonal and player1 will win in one move to the angle. So player2 has only one another opportunity: to move the bishop to the first shape board. Then like in previous paragraph player1 will win.

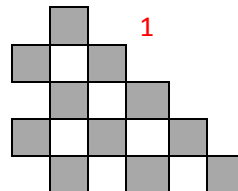
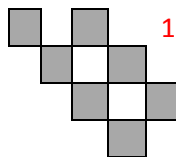
- 6) If he moves to the board with the second shape player1 will also win in one move to the end of the diagonal line.
- 7) So player1 wins in Situation 1.
- 8) If player2 does not make his all next moves to the neighbor diagonal line to the line that player1 made his previous move along, then the situation becomes similar to the Situation 2.

Situation 2:

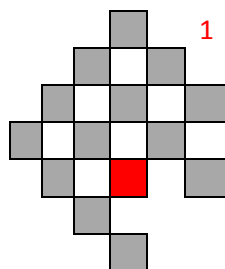
- 1) If player2 in his first move has not gone to the neighbor diagonal line to the line that player1 made his first move along, player1 would pass this diagonal line to the end moving in opposite direction to his first move's direction. The board is divided into two pieces: one contains the square on which the bishop was originally placed ('beginning square'), the other does not contain it.
- 2) If player2 makes his second move to the side that contains the beginning square, player1 would pass this diagonal line to the end moving in direction to his first move.
- 3) If player2 has in his first move gone two squares and in second only one, player1 would win by this turn. Example:



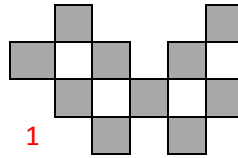
- 4) If player2 made his second move to the diagonal line that contains the beginning square, player1 would pass this diagonal line to the end moving in direction to his first move. Now player2 can go to two board having these shape:



- 5) On the first board player1 wins by following the main rule. On the second board player1 wins like in the Situation 1.
- 6) If player2 makes his second move some other way, but on the side of the board that contains the beginning square, player1 would pass this diagonal line to the end moving in direction to his first move. Now player2 can not go anywhere, so he loses, or he can go to only one shape-board:



- 7) Player1 wins by following the main rule.
- 8) If player2 makes his second move to the side of the board that does not contain the beginning square, player1 would pass this diagonal line to the end moving in direction to his first move.
- 9) Player2 will be able only to go to a board having shapes like the first shape in Situation 1 (like in Situation 1 player1 wins) or this shape:



- 10) Player1 wins by following the main rule.
- 11) So player1 wins in Situation 2.

To sum up:

- 1) Wins the player, who is the first to move the bishop, if he follows the strategy rules.