# International tournament of Young Mathematicians

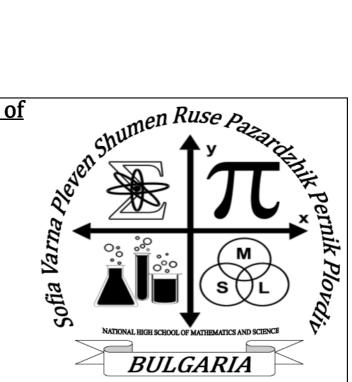
24<sup>th</sup> June – 1<sup>st</sup> July 2010 University Paris-Sud 11, Orsay, France

### Team:

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#### Problem:

9. A Topological Problem



# **Abstract**

Problem 9. is examined in details. The following results are reached:

9.1. Answer: 3

9.2. a) It is shown that int(conv(A)) = int(conv(cl(A))) holds true. 9.2.

b) Answer: 8

9.3. Answer: 8.

We used a method for the solution of this problem which exhausts all possibilities. Therefore, this method exhausts all cases of the problem.

## **Theoretical Part**

- 1. Closed set, without the points on the boundary, is an open set.
- 2. *Interior* the operation removes the boundary of the set (these objects from the set, which have less than *n* dimensions disappear).
- 3. *Closure* adds the boundary of the set.
- 4.  $Convex\ cover\ -$  for a set of points A is the minimal convex set containing A.

## **Solution**

#### Problem 1.

*A* consists closed and also opened sets (and some more, which are neither opened, nor closed).

int(A) and cl(A) change only the boundary of A.

But A, int(A) and cl(A) are different.

$$int(int(A)) = int(A)$$
  
 $int(cl(A)) = int(A)$   
 $cl(int(A)) = cl(A)$   
 $cl(cl(A)) = cl(A)$ 

No matter how the operations are arranged, we obtain int(A) or cl(A) (only if we have made at least one operation).

I.e. the sets are A, int(A) i cl(A) - 3 sets.

Example: A = [0; 1)

#### Problem 2.

a)

$$int(conv(A)) \neq \emptyset \Rightarrow$$

We can contend that the convex cover of A has area because int(A) removes the boundary and we obtain something more than empty set.

conv(A) and conv(cl(A)) [eventually] are different only by their boundary (because if A is closed, cl(A) doesn't change it) and when we apply int(conv(A)) and int(conv(cl(A))), we remove the boundary (if it exists) and consequently obtain the same set, i.e. int(conv(A)) = conv(A)

$$int(conv(cl(A)))$$
 - Q.E.D.

b)

If 
$$int(conv(A)) = \emptyset$$
,

Then, since *int* removes the boundary, we have that  $conv(A) \in \mathbb{R}$ , i.e. we return the case to the previous problem, but with the operation conv.

Then, the possible sets are A, int(A), cl(A)

If  $int(conv(A)) \neq \emptyset$ , we have:

(the operations only with *int* and *cl* follow directly by the previous problem)

- 1) int(int(int(A))) = int(A) [from the previous problem];
- 2) int(int(cl(A))) = int(A) [from the previous problem];
- 3) int(int(conv(A))) = int(conv(A)), because int(int(B)) = int(B);
- 4) int(cl(int(A))) = int(A) [from the previous problem];

- 5) int(cl(cl(A))) = int(A) [from the previous problem];
- 6) int(cl(conv(A))) = int(conv(A)), because int(cl(B)) = int(B);
- 7) int(conv(int(A))) = conv(int(A)), because after int(A) the convex cover cinv(int(A)) don't have a boundary to be removed;
- 8) int(conv(cl(A))) = int(conv(A)), because even we add the boundary with cl, after that int removes it;
- 9) int(conv(conv(A))) = int(conv(A)), because conv(conv(B)) = conv(B);
- 10) cl(cl(int(A))) = cl(A) [from the previous problem];
- 11) cl(cl(cl(A))) = cl(A) [from the previous problem];
- 12) l(cl(conv(A))) = cl(conv(A)), because cl(cl(B)) = cl(B);
- 13) cl(int(cl(A))) = cl(A) [from the previous problem];
- 14) cl(int(int(A))) = cl(A) [from the previous problem];
- 15) cl(int(conv(A))) = cl(conv(A)), because cl(int(B)) = cl(B);
- 16) cl(conv(int(A))) = cl(conv(int(A))) [from the previous problem];
- 17) cl(conv(cl(A))) = cl(conv(A)), because never mind, in the end of the operations we take the boundary of A;
- 18) cl(conv(conv(A))) = cl(conv(A)), because conv(conv(B)) = conv(B);

- 19) conv(conv(int(A))) = conv(int(A)), because conv(conv(B)) = conv(B);
- 20) conv(cl(A)) = conv(cl(A)), because conv(conv(B)) = conv(B);
- 21) conv(conv(A)) = conv(A), because conv(conv(B)) = conv(B);
- 22) conv(int(int(A))) = conv(int(A)), because int(int(A)) = int(A);
- 23) conv(int(cl(A))) = conv(int(A)), because int(cl(A)) = int(A);
- 24) conv(int(conv(A))) = int(conv(A)), because conv dava gives convex cover, int removes the boundary and after that we take the convex cover, i.e. after conv and int one more time conv doesn't change the set;
- 25) conv(cl(int(A))) = conv(cl(A)), because cl(int(A)) = cl(A);
- 26) conv(cl(cl(A))) = conv(cl(A)), because cl(cl(A)) = cl(A);
- 27) conv(cl(conv(A))) = cl(conv(A)) analogical to 24);

Consequently the possible outcomes are the following:

$$int(A)$$
 $int(conv(A))$ 
 $conv(int(A))$ 
 $cl(A)$ 
 $cl(conv(A))$ 
 $cl\left(conv(int(A))\right)$ 
 $conv(cl(A))$ 

## conv(A)

Answer: 8.

#### Problem 3.

The answers for n > 2 are analogical to the previous problem (9.2.b) because the operations int, cl and c conv work only with the boundary. Whatever operations we apply (it doesn't matter the order) we get the same sets as in the previous problem (9.2.b).

Answer: 8.