International tournament of Young Mathematicians

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Team:
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Problem:
5. A Strange Network

Abstract

We have solved and proved 5.1.

The obtained formula is: $T_{min} = \binom{n-2}{k} + 2.\binom{n-2}{k-1} + 1.$

We have examined the suggested cases; also when $k = 2$, and the general case.

In 5.2. we have proved that for $k > 1$ we can always find out the primary code and we have given a hypothesis about the value of T_{min}

Solution

Problem 1:

The numbers are different.

1) $\boldsymbol{k = n - 1}$

With the first $k = n - 1$-subcode we find out $n - 1$ from all the n numbers, and also their order in the primary code.

With the second $k = n - 1$-subcode we find out the n-th number and its position in the primary code. If the missing number in the second k-subcode is neighbor by position with the missing number in the first k-subcode, then we can't define the primary code, because we don't know which number is with the smaller index.

With the third k-subcode the problem is solved, because by the conditions, all the subcodes are different, i.e. the missing number here won't be missing in the previous two subcodes.

Consequently, with three $(n-1)$-subcodes we can find out the primary code i.e. $T_{min} = 3$.

2) $\boldsymbol{k = n - 2}$

We can consider that the k-subcode sends the numbers from a_1 to a_{n-2} (for concreteness: Carl doesn't know that he receives exactly these elements of A).

With $k = n - 2$ more subcodes we send all the codes where we have a_{n-1} and $k - 1$ from the first $n - 2$ numbers.

With $n - 2$ more subcodes we send all the codes, where we have a_n and $k - 1$ from the first $n - 2$ numbers.

Till now we have send $1 + n - 2 + n - 2 = 2n - 3$ subcodes and these are all the subcodes where a_{n-1} and a_n doesn't participate simultaneously.

We know the exact order of the first $n - 2$ numbers, but we can't be sure about the last two - if they are $a_{n-1}, a_n,$ or a_n, a_{n-1}.

Next subcode will surely contain a_{n-1}, and a_n, which will define their order.

Consequently, we need $2n - 3 + 1 = 2n - 2$ subcodes, i.e. $T_{min} = 2n - 2$.

3) $\boldsymbol{k = \left[\frac{n}{2}\right]}$

3.1) $\boldsymbol{n = 2m}$, i.e. $\boldsymbol{k = m}$

Again without loss of generality (for concreteness, without adding more symbols) we can consider that firstly we send the first numbers and then – the next ones (Carl doesn't know that).

Then with $\binom{2m-2}{m}$ subcodes we can understand the first $2m-2$ and their exact order.

After that with $\binom{2m-2}{m-1}$ more we can send all the subcodes, including a_{2m-1} and $m-1$ from the first $2m-2$ numbers.

Analogically for a_{2m}.

Till now, we have send all the subcodes, which either include neither a_{2m-1}, nor a_{2m}, or include only one of them.

Analogically to the previous case: we know the exact order of everything except if the last two numbers are a_{2m-1}, a_{2m} or a_{2m}, a_{2m-1}.

We can find out that with the next subcode, which surely includes the missing two numbers.

Then $T_{min} = \binom{2m-2}{m} + 2.\binom{2m-2}{m-1} + 1$.

3.2) $\boldsymbol{n = 2m + 1}$, i.e. $\boldsymbol{k = m}$

Analogically to the previous case we have that the subcodes, which don't include the last two numbers are $\binom{2m-1}{m}$; the subcodes, which include exactly one from the last two numbers are $\binom{2m-1}{m-1}$ and with one more move we find out the order of the primary code.

I.e. $T_{min} = \binom{2m-1}{m} + 2.\binom{2m-1}{m-1} + 1$

4)

In the common case for $k > 1$ (if $k = 1$ with n subcodes we can't find out the exact order of the numbers) analogically to the other cases we have:

$\binom{n-2}{k}$ – all the subcodes, which don't include the last two numbers;

$\binom{n-2}{k-1}$ – all the subcodes, which don't include one from the two last numbers;

$\binom{n-2}{k-1}$ – all the subcodes, which include the other from the last two numbers;

1 – one more subcode, which includes the last two numbers.

I.e. $T_{min} = \binom{n-2}{k} + 2.\binom{n-2}{k-1} + 1$.

Note: Lets mark that in the case $k=2$, T_{min} is the number 2-subcodes, which can be send ($T_{min}=\binom{n}{2}$).

Note 2: Because the numbers are different, it is necessary $\alpha \geq n-1$, for having at least n different values for the a_i-s

5) $\alpha = 1$

In this case the numbers are different.

Consequently $n=1$ or $n=2$.

When $n=1$ follows that $k=1$ and we need only one subcode, i.e. $T_{min}=1$.

When $n=2$ and $k=2$ again with one subcode we can find out the exact order of the prime code, i.e. $T_{min}=1$.

When $n=2$ and $k=1$ we can't find out surely whether the code is 0,1 or 1,0.

Problem 2:

The numbers can also be equal.

If $k=1$, analogically to the previous, by n subcodes we can find out the n numbers, but not their order.

If $k>1$, certainly with $\binom{n}{k}$ subcodes (these are the all k-subcodes) we can find out the exact order.

For example, if the code is compounded only of equal numbers, we will need $\binom{n-1}{k}+1$ subcodes for finding out the primary code - $\binom{n-1}{k}$ are the all subcodes, which are compounded of k in number from the $n-1$ numbers, and the last subcode includes the n-th number.

If in any moment in the subcodes appears another number, then the needing number T_{min} will change in dependence of how many different values the a_i-s take, and also in dependence of that how many consecutive equal elements does the primary code have.

The case when $k=n-1$ is analogical to the case, where the numbers can't be equal.

The case when $k=2$, is analogical again to the case where the a_i-s are different.

When $\alpha=1$, i.e. in the primary code there are only 0 or 1:

If $k=2$, we have an easy way to find out the code: counting how many times we have the subcode $(0;0)$. This number must be like $\binom{x}{2}$, where x is the number of nulls in the primary code. Then $n-x$ is the number of ones and in dependence of the subcodes $(0;1)$ and $(1;0)$ (their number; by analogical way) we can find out their exact order.