International tournament of Young Mathematicians

24th June – 1st July 2010
University Paris-Sud 11, Orsay, France

Team:
Bulgaria

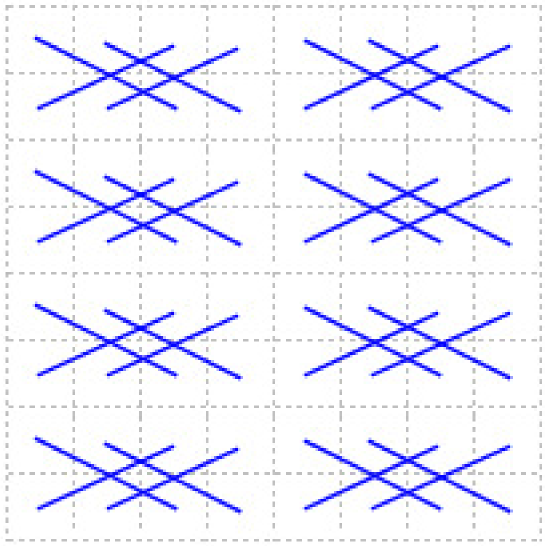
Problem:
4. A Baby Chess

Abstract

The solution examines the movement of the knight without leaving a trace. An algorithm for creating a winning strategy is constructed but proofs are given for only a part of the cases. Studying the cases without proofs, major ideas have been developed. All these facts can be very useful for further research of the strategies.

Solution

A Baby Chess



1.

On the figure it is shown the $m = n = 8$ case. If the first player plays on the blue lines, he will win the game.
Analogically with tables $4k \times 2l$ if we divide it by $k \times l$ rectangles 4×2.
It is obvious that all the board can be divided by such rectangles. That's why each square is is only one of those 2×4 rectangles. In this case the first player wins.

If the board is $(2s + 1 \times (2t + 1)$:
There exist a square without a segment or with two segments. If the first player plays on it, the second player has a winning strategy.

Consider the next case: (2s+1)×(2t+1) board.
Then there is a square with no line or with 2 lines. If there is such a square and the first student makes his move on it the second will start to move on the lines. The student with winning strategy depends from the number of this squares and how many shifts are made.

Consider the case with small m and n (m≤n):

1) m=1

There can't be made any move. So the first student lose and the second wins.