

LCME, Russia, Saint-Petersburg, Problem 6.

1) Can the pattern

$2134n-1 \ n$

$2345n \ 1$

be obtained from the pattern

$2314n \ n$

$2345n \ 1$

using the operations A and B?

Let's equate operation A this way: it compares to $(\alpha, \beta)(\alpha\beta, \beta)$, where α, β – some rearrangements of $1 \dots n$. B compares to $(\alpha, \beta)(\alpha, \beta\alpha)$.

Let's equate $2 \ 3 \ 4 \ 1 \dots n-1 \ n$ to a rearrangement $(1 \ 2); 2 \ 3 \ 4 \dots n \ 1$ to rearrangement $(1 \ 2 \dots n); 2 \ 3 \ 1 \ 4 \dots n$ – to $(1 \ 2 \ 3)$.

(?) $((1 \ 2 \ 3), (1 \ 2 \dots n)) \rightarrow ((1 \ 2), (1 \ 2 \dots n))$.

B changes only the parity of second rearrangement. B^*B , operating on the second component of the pair, is 1 of group of rearrangements. So using B is not useful.

We can get rearrangement, in which 1 turns to 2, by operation A only using it n times. But $A^*A^* \dots^*A$ n times, is 1 of group of rearrangements and first component won't change. So we can't obtain second pattern from first using A and B.