LCME, Russia, Saint-Petersburg, problem 3 “monotonic squares”

1. There is very nice example in the text of this problem, but the first element of this infinite family of increasing squares should be $16 = 4^2$. Because $16\sim < 1156$
   Basing on it it’s possible to easily find another same simple example of infinite family of increasing squares: $5^2 = 25\sim < 35^2 = 1225\sim < 335^2 = 112225\ldots$ etc.
   $\left\{a_i | a_i = \overline{\underbrace{3\ldots3}_{\iota-1}5}\right\}_{i\ni N}$ and the entire family of squares is $\left\{b_i | b_i = \overline{\underbrace{1\ldots1}_{\iota}\underbrace{2\ldots2}_{\iota+1}5}\right\}_{i\ni N}$

   Another example is $\left\{a_i | a_i = \overline{\underbrace{3\ldots3}_{\iota}7}\right\}_{i\ni N}$ and the entire family os squares is $\left\{b_i | b_i = \overline{\underbrace{1\ldots1}_{\iota}35\underbrace{6\ldots6}_{\iota+1}9}\right\}_{i\ni N}$

   However, any generalization of the problem is too complicated.

2. Yes, there exist infinite families of decreasing squares. And, if we suppose that there exists infinite set of not ordered decreasing squares, then there exist infinite set of infinite families of decreasing squares. The simplest example of such family is $\{a_i | a_i = 10^{2i}\}_{i\ni N}$
   But, using this method(multiplying by 10 each time) we can make an infinite family of decreasing squares from any decreasing square, for example : $9^2=81$ and infinite family of it is $\left\{a_i | a_i = \overline{81\underbrace{0\ldots0}_{2(\iota-1)}}\right\}_{i\ni N}$

3. Unfortunately, we haven’t solved this part of problem, but we can’t skip this park in this text because we’ve solved 4th part and our MS word 2007 do not allow to put number 4 to the list without putting number 3. By the way, one additional point for the sense of humor would be enough…

4. Any maximal family of decreasing squares is infinite, because if we can’t find next non-trivial element of the family we can always add “0” to the right of the number (as it’s shown in the part 2). So there are no maximal families of decreasing squares.