

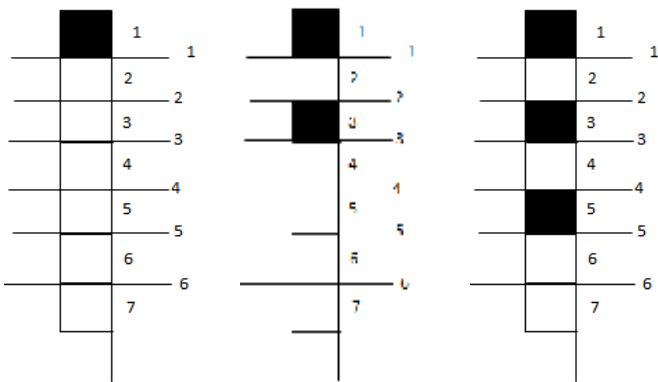
1) Optimal variant is colouring as a chess desk :

2) Lets prove that for any $n \times 1$ rectangle optimal variant is $\frac{n}{2}$ if n is even, and $\frac{n+1}{2}$ if n is odd.

Proof.

Consider first line. There is only two squares which are symmetric with respect to it: the first and the second. And we can paint only one of them

a) If we paint the first square. Then consider the second line. There is only two pairs of squares which are symmetric with respect to this line. It is squares 1-4 and 2-3. Find out that squares above the second line(1 and 2) are uniquely defined, then squares 3 and 4 must be painted(not painted) if and only if square symmetric to them is not painted(painted).

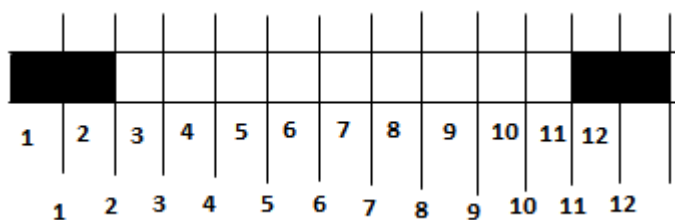


Similarly for the third line. All square above it are uniquely defined, thus they uniquely define squares which are symmetric to them. And similarly for any line. It is easy to notice that in this case result is $\frac{n}{2}$ if n is even and $\frac{n+1}{2}$ if n is odd.

b) If we paint the second square. In this case make similar actions. Then if n is even nothing is change. But if n is odd then result will be less by one.

Now consider $n \times m$ square, paint first column as we did in the first point. Cosider first vertical line, to the left of it, all the squares are defined, then symmetrical to them squares to the right will have opposite colouring, the similarly for the second and for all the following. It is obvious that dissymmetric with respect to all horizontal lines will stay

2) At first consider $n \times 1$ rectangle . Let's consider the first line. There is only one pair of squares(1 and 2) which are symmetric with respect to this line. Similarly for the last line. Now we have symmetry with respect to the lines 1,6,7,12. (pic 1). Let's consider third the line. We can paint only third or fourth square. If we paint third square we add symmetry with respect only to the line 2. But if we paint fourth square we add symmetry with respect to lines 2,8. So it is better to paint forth square, because we gain more lines with symmetry. Then consider third line. We can paint fifth or sixth



square. If we paint fifth one we gain only one line with symmetry. But if we paint sixth we gain two lines with symmetry. So it's better to paint sixth square. Similarly for any

line .s