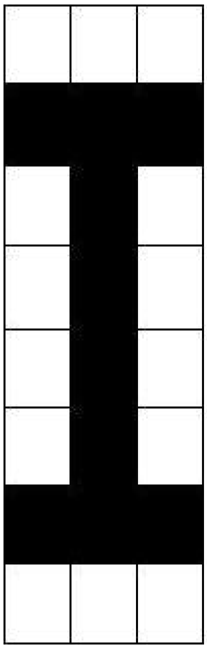
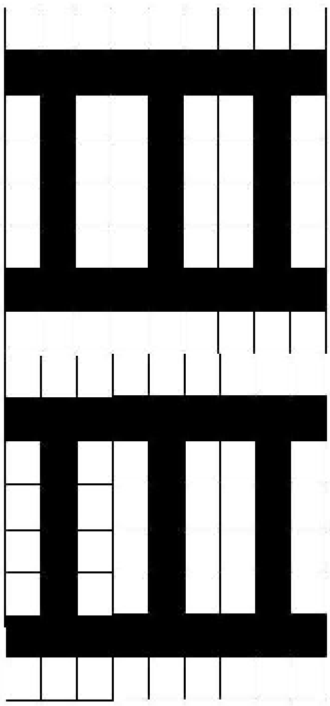
Placements of Pentominoes

Question 1:

To give a upper bound to *S(m,n),* we found an example of paving :

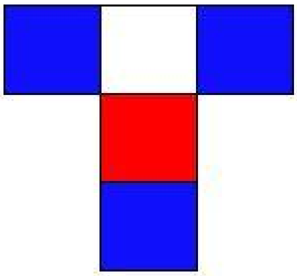


We can pave a *m*n* rectangle with this placement. For example :



The paving is done by a $3 * 8$ rectangle containing 2 pentominoes

Then we have
$S(m,n) < 2mn/(3*8) = mn/12$
For the lower bound :

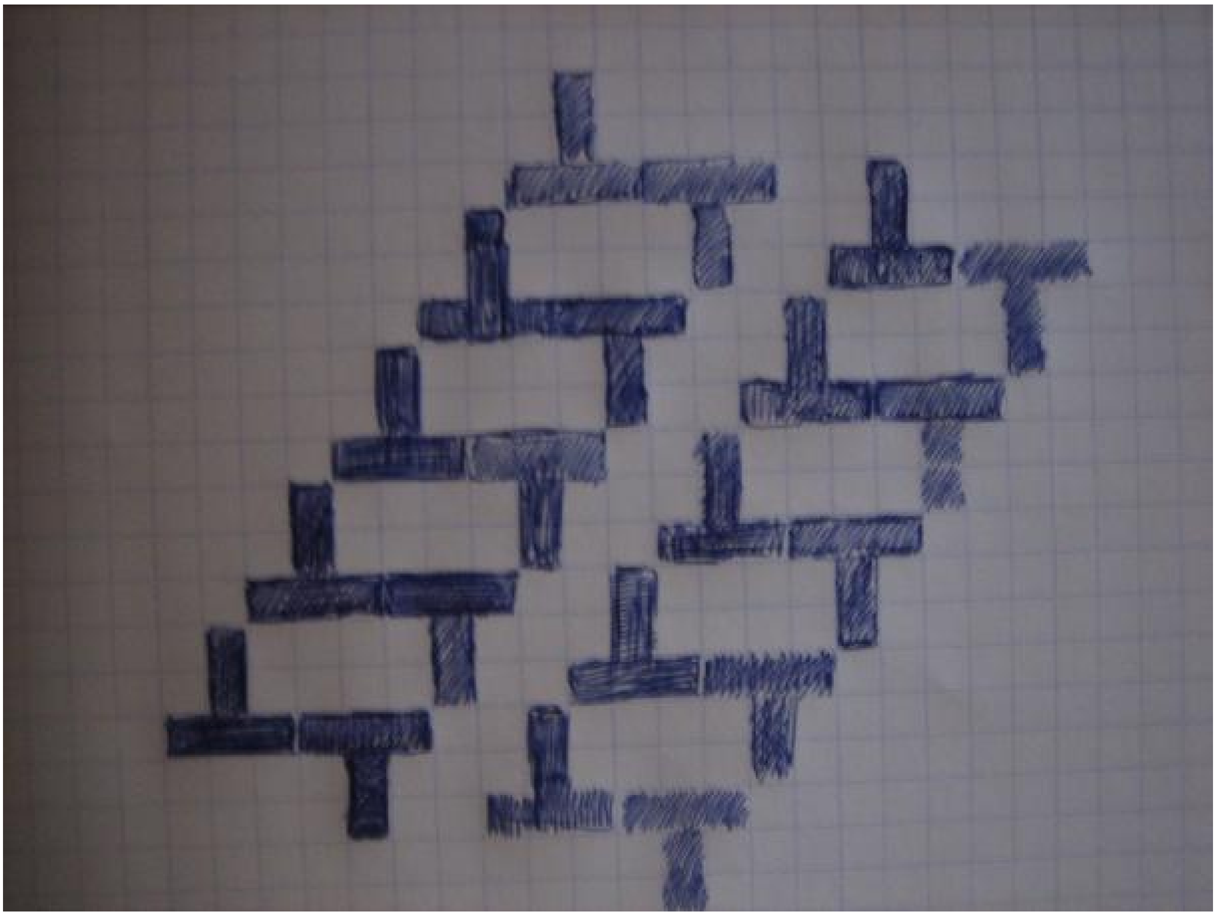


Let $s1$ be the number of blue cells in the $m*n$ rectangles, $s2$ the number of red cells and $s3$ the number of white cells.
Let y the number of free cells.
For any red cell, we have at least 3 blue cells around. (a cell A is around another cell B if A is one of the 8 cells of the square of centre A).
For any white cell, we have at least 2 blue cells around.
For any free cell, we have at least 2 blue cells around.
At worst, we count a blue cell 8 times.
Finally, we obtain :
$S1>(3s2+2s3+2y)/8$ **(1)**
But, obviously, we have $s2=s3=s1/3$ and $y=mn-5s1/3$.
Let N the number of pentominoes in the rectangle.
We have $N=5s1/3$.
Using the inequality **(1)**, we obtain $N>2mn/29$.
Then $S(m,n)>2mn/29$.

Finally,

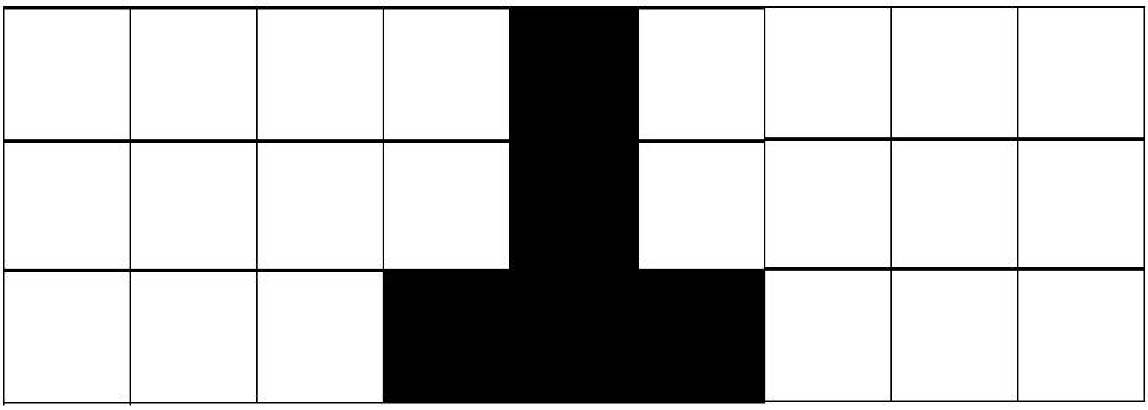
$$2mn/29<S(m,n)<mn/12$$

Note : we can improve the upper bound (1/12,5) with this example :



2)

We just know that there exist a winning strategy for one of the players,
If m=3 and n is odd (n>3), we proved that the first player has a winning strategy,
Indeed, he just has to put his first pentomino at the center like this :



Then the first player just has to play the symmetric (with respect to the vertical axis of symmetry of the rectangle) of the pentomino which has just been putted by the second player,
Clearly it is a winning strategy for him,